Cauer Circuit Representation of Homogenized Eddy-Current Field Based on Legendre Expansion in Magnetic Sheet

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Using the Legendre expansion of magnetic field distribution, this article derives the standard Cauer circuit representation of the frequency-dependent properties of magnetic sheets and discusses the physical meaning of the standard Cauer circuit. The Cauer circuit is applied to the dynamic hysteresis modeling of silicon steel under the PWM excitation.

***Index Terms*—Cauer realization, dynamic hysteresis, Legendre polynomial, pulse width modulation.**

I. Introduction

PROGRESS in the semiconductor technology achieves the efficient power control with high frequency switching operation, which induces complex dynamic hysteretic magnetic fields in iron cores, where minor hysteresis loops and eddy-current fields with thin skin-depth are significant.

Several homogenization methods [1]-[3] have been developed for the efficient analysis of laminated cores avoiding finite-element division along the stacking direction of silicon steel sheets. However, accurate evaluation of the eddy-current field in the steel sheet having nonlinear magnetic properties is difficult without one-dimensional sub-analysis along the stacking direction [1], [2].

A linear eddy-current theory derives the standard and physical Cauer circuit representations [3]-[5] of the frequency-dependent properties of magnetic sheets. While the standard Cauer circuit is obtained directly from the linear theory, the physical Cauer circuit has been studied for the nonlinear analysis because its physical meaning is clear. In the nonlinear case, however, the physical Cauer circuit requires more inductive elements than expected from the linear circuit, even after circuit optimization [3].

Deriving the standard Cauer circuit from the Legendre expansion of magnetic field, this study discusses the physical meaning of this circuit and applies it to the dynamic hysteresis modeling of steel sheet.

II. Derivation of Cauer Circuit by Legendre Expansion

A. Cauer realization

The magnetic field in the steel sheet is governed by

$$\partial^2 H/\partial z^2 = \sigma \partial B/\partial t \tag{1}$$

where σ is the conductivity. A linear eddy-current theory for the magnetic sheet gives the relation between the average magnetic flux density B_{av} and the surface magnetic field H_{s} as

$$B_{\mathrm{av}} / H_{\mathrm{s}} = \mu\,(2/kd)\tan(kd/2) = \mu\,(2/\mathrm{j}kd)\tanh(\mathrm{j}kd/2) \tag{2}$$

where $k = (-\mathrm{j}\omega\sigma\mu)^{1/2}$, ω is the angular frequency, μ is the permeability, and d is the sheet thickness. By expanding $\tan(kd/2)$ or $\tanh(\mathrm{j}kd/2)$, the relation (2) is represented by the infinite RL ladder circuit [4], [5] as is shown in Fig. 1, where μ and $4/\sigma d^2$ are replaced by the inductance L and resistance R, respectively. This circuit is called standard Cauer circuit.

B. Legendre expansion

Ref. [2] developed a homogenization method using the Legendre expansion where the magnetic flux density distribution along the thickness direction is expanded by the Legendre polynomials P_{2n} (n = 0, 1, …) as

$$B(t,z) = b_0(t)P_0\left(\frac{2z}{d}\right) + b_2(t)P_2\left(\frac{2z}{d}\right) + b_4(t)P_4\left(\frac{2z}{d}\right) + \cdots \,. \tag{3}$$

This subsection derives the standard Cauer circuit using (3).

By defining

$$\boldsymbol{P} = [P_0(2z/d),\, P_2(2z/d),\, P_4(2z/d),\, \ldots]^{\mathrm{T}} \tag{4}$$

$$\boldsymbol{h} = [h_0(t),\, h_2(t),\, h_4(t),\, \ldots]^{\mathrm{T}} = (1/\mu)[b_0(t),\, b_2(t),\, b_4(t),\, \ldots]^{\mathrm{T}} \tag{5}$$

(1) is rewritten as

$$\boldsymbol{h}^{\mathrm{T}} \frac{\partial^2 \boldsymbol{P}}{\partial z^2} = \sigma\mu \frac{\mathrm{d}\boldsymbol{h}^{\mathrm{T}}}{\mathrm{d}t} \boldsymbol{P} \,. \tag{6}$$

Using the relations,

$$\frac{\mathrm{d}P_{2n}(x)}{\mathrm{d}x} = (4n-1)P_{2n-1}(x) + (4n-5)P_{2n-3}(x) + \cdots + 3P_1(x)\,,$$
$$\frac{\mathrm{d}P_{2n+1}(x)}{\mathrm{d}x} = (4n+1)P_{2n}(x) + (4n-3)P_{2n-2}(x) + \cdots + P_0(x) \tag{7}$$

(6) is written as

$$\left(\sigma\mu \frac{\mathrm{d}\boldsymbol{h}^{\mathrm{T}}}{\mathrm{d}t}\boldsymbol{F} - \frac{4}{d^2}\boldsymbol{h}^{\mathrm{T}}\boldsymbol{G}\right)\boldsymbol{Q} = 0 \tag{8}$$

where

$$\boldsymbol{Q} = [P_0(2z/d),\, 5P_2(2z/d),\, 9P_4(2z/d),\, \ldots]^{\mathrm{T}} \tag{9}$$

$$\boldsymbol{F} = \mathrm{diag}\begin{bmatrix}1 & 1/5 & 1/9 & 1/13 & \cdots\end{bmatrix} \tag{10}$$

$$\boldsymbol{G} = \begin{bmatrix} 0 & 0 & 0 & 0 & \cdots \\ 3 & 0 & 0 & 0 & \cdots \\ 3+7 & 7 & 0 & 0 & \cdots \\ 3+7+11 & 7+11 & 11 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{11}$$

From (8) and the orthogonality of $P_{2n}(2z/d)$ in $[-2/d, 2/d]$,

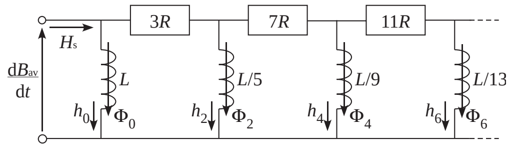

Fig. 1. Standard Cauer circuit.

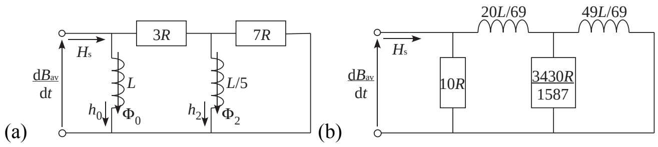

Fig. 2. (a) Standard and (b) physical Cauer circuits.

$$\mu \boldsymbol{F}\frac{\mathrm{d}\boldsymbol{h}}{\mathrm{d}t}-\frac{4}{\sigma d^2}\boldsymbol{G}^{\mathrm{T}}\boldsymbol{h}=L\begin{bmatrix}1 & 0 & 0 & 0 & \cdots\\ 0 & 1/5 & 0 & 0 & \cdots\\ 0 & 0 & 1/9 & 0 & \cdots\\ 0 & 0 & 0 & 1/13 & \cdots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{bmatrix}\frac{\mathrm{d}\boldsymbol{h}}{\mathrm{d}t}$$

$$-R\begin{bmatrix}0 & 3 & 3+7 & 3+7+11 & \cdots\\ 0 & 0 & 7 & 7+11 & \cdots\\ 0 & 0 & 0 & 11 & \cdots\\ 0 & 0 & 0 & 0 & \cdots\\ \vdots & \vdots & \vdots & \vdots & \ddots\end{bmatrix}\boldsymbol{h}=0 \tag{12}$$

is obtained. Since

$$H_s(t)=\frac{1}{\mu}B(t,\frac{d}{2})=h_0(t)+h_2(t)+h_4(t)+\cdots \tag{13}$$

(12) describes the state equation of the standard Cauer circuit shown in Fig. 1.

When the frequency is low, $h_0(t)$ ($\approx H_s(t)$) is the dominant current and the flux change $\mathrm{d}\Phi_0/\mathrm{d}t = L\mathrm{d}h_0/\mathrm{d}t$ induces the eddy-current $h_2(t) \approx (\mathrm{d}B_{av}/\mathrm{d}t) / (3R)$. Accordingly, the magnetic flux $\Phi_2 = (L/5)h_2$ is regarded as the secondary flux generated by the induced current h_2. The physical meaning of standard Cauer circuit above, discussed in [6], is supported by the Legendre expansion. The uniformly distributed magnetic flux density b_0P_0 induces the eddy-current distributed linearly along the z-direction, which yields the parabolically distributed b_2P_2.

By truncating the standard Cauer circuit as shown in Fig. 2(a), it can be converted equivalently to another RL ladder circuit as shown in Fig. 2(b). This is called the physical Cauer circuit because the ratio of the inductances corresponds to the nonuniform physical division [3]-[5] of the half thickness $d/2$. The equivalence of truncated standard Cauer circuit to the homogenization method in [2] will be proven in the full paper.

C. Nonlinear inductors

When the static magnetic property of a steel sheet has nonlinearity represented by $H_s = h_{DC}(B_{av})$, the inductor L is replaced by the relationship $h_0 = h_{DC}(\Phi_0)$. Magnetic fluxes Φ_2, Φ_4, … can be regarded as corrections to flux Φ_0. If the flux correction is small, it holds that

$$H_s = h_{DC}(\Phi_0+\Delta\Phi) \approx h_{DC}(\Phi_0) + (1/\mu_d)\Delta\Phi \tag{14}$$

where $\mu_d = [\mathrm{d}h_{DC}(\Phi_0)/\mathrm{d}\Phi_0]^{-1}$ is the differential permeability. Replacing L by μ_d, the relation between h_{2n} and Φ_{2n} (n = 1, 2, …) is approximated as

$$h_{2n} = (4n+1)\,\Phi_{2n} / \mu_d\,. \tag{15}$$

III. Numerical Result

The standard Cauer circuit shown in Fig. 2(a) is applied to the dynamic hysteresis modeling of silicon steel sheet under the PWM excitation. The static hysteretic property $H = h_{DC}(B)$ is represented by the play model [7]. The fundamental and carrier frequencies are 50 Hz and 5 kHz. The former frequency component is not greatly affected by elements of $L/5$ and $7R$ whereas the latter component is affected by them and yields small minor hysteresis loops. Accordingly μ_d can be given by the incremental permeability of minor loops, which is roughly approximated by $\mu_{rev} = \mathrm{d}h_{rev}(B)/\mathrm{d}B$ (see Fig. 3) where $h_{rev}(B)$ is the reversible component of $h_{DC}(B)$. Fig. 4(a) shows the simulated BH loops using a hysteretic inductor for the element L and a non-hysteretic inductor with $\mu_{rev}/5$ for $L/5$, which agree with measured loops. In contrast, the classical eddy-current theory overestimates the component of carrier frequency as shown in Fig. 4(b). The representation of $L/5$ (, $L/9$, …) will be further discussed in the full paper, where the comparison with the finite element eddy-current analysis and the physical Cauer circuit representation will be presented.

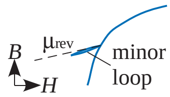

Fig. 3. Permeability for minor BH loop.

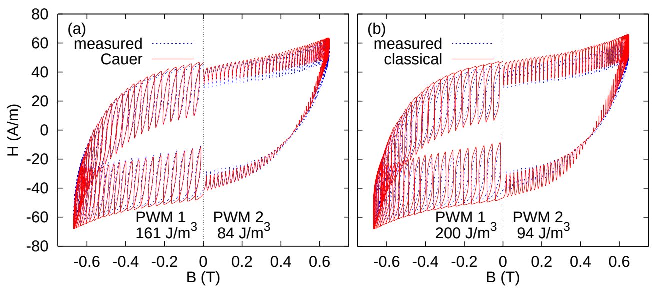

Fig. 4. Simulated BH loops for 2 types of PWM wave forms where measured iron losses per cycle are 162 and 83 J/m^3: (a) Cauer circuit and (b) classical eddy-current theory.

IV. References

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